

## Homework # 2

Due Thursday, April 27, 2017, at 2:30 PM

*Collaboration and discussions are not allowed on Problem 1  
and are allowed and encouraged on Problem 2*

### 1. Positional Number Systems (*Collaboration is not allowed on this Problem*)

Recall that an integer  $N$  represented in base- $b$  is denoted by  $N = (d_m, d_{m-1}, \dots, d_1, d_0)_b$  with

$$N = \sum_{i=0}^m d_i \cdot b^i$$

and  $0 \leq d_i \leq b - 1$ .

- (a) Given  $X$  and  $Y$ , two numbers with  $m+1$  digits, represented in a base- $b$  positional system. Namely,  $X = (x_m, x_{m-1}, \dots, x_1, x_0)_b$  and  $Y = (y_m, y_{m-1}, \dots, y_1, y_0)_b$ . Prove the following statements for any base  $b$ .
- $X = Y$  if and only if  $x_i = y_i$  for all  $0 \leq i \leq m$ .
  - Assume that  $X \neq Y$ . Suggest a simple algorithm to decide which number is bigger. *Hint:* Let  $j$  be the most significant position where  $X$  and  $Y$  differ. Prove that  $X > Y$  if and only if  $x_j > y_j$ .

*Reminder:* Recall that an 'if and only if' (in short iff) proof requires proving the two directions of the claim; for instance, if we want to prove  $(A \text{ iff } B)$ , then we must prove  $(\text{If } A, \text{ then } B)$  and  $(\text{if } B, \text{ then } A)$ .

- (b) Prove the claims in (a) for  $X$  and  $Y$  two numbers with  $m+1$  digits, represented in a base- $b$  no-0 positional system. *Before attempting this problem, it might be helpful to solve Problem 2.*
- (c) Prove the claims in (a) for  $X$  and  $Y$  two numbers with  $m$  digits, represented in a factoradic number system. *Before attempting this problem, it might be helpful to solve Problem 2.*
- (d) What are all the possible solutions for  $b$ ? Show your work.
- $(1234)_b + (5432)_b = (6666)_b$
  - $(41)_b \div (3)_b = (13)_b$
  - $\sqrt{(41)_b} = (5)_b$
- (e) What are all the possible solutions for  $b$  and  $c$ ? Show your work.
- $(121)_b = (100)_c$
  - $(50)_b = (43)_c$

## 2. Strange and Beautiful Number Systems (*Collaboration is allowed on this Problem*)

### (a) Base- $b$ no-0 Positional Number System

In class we studied the base-10 no-0 positional system where a positive integer  $N$  is represented as follows:

$$N = \sum_{i=0}^m d_i \cdot 10^i$$

with  $d_i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, A = 10\}$  for all  $0 \leq i \leq m$ .

Examples,  $N = (10)_{10} = (A)$ ,  $N = (100)_{10} = (9A)$  and  $N = 0 =$  (the blank string).

We expect you to solve the following problems with syntax-based procedures (as was demonstrated in class) and not by base-conversion procedures.

- i. What is the base-10 no-0 representation of  $(30006700801)_{10}$ ? Use the algorithm presented in class.
- ii. Suggest an algorithm to convert from base-10 no-0 to base-10. Use your algorithm to find the base-10 representation of  $(AA21A36A99A)$ ?
- iii. Suggest an analogous base-2 no-0 representation.

*Hint: the number  $(4)_{10}$  is represented by  $(100)_2$  in base-2 and by  $(12)$  in the base-2 no-0 representation.*

Apply the analogous algorithms from parts (i) and (ii) to represent the base-2 number  $(10110100100110110)_2$  in base-2 no-0 representation; and to represent the base-2 no-0 number  $(1122112121121112)$  in the base-2 representation.

### (b) Factoradic Number System

A number  $N$  represented in the factoradic system is denoted by  $N = (a_m, a_{m-1}, \dots, a_2, a_1)_!$  with

$$N = \sum_{i=1}^m a_i \cdot i!$$

and  $0 \leq a_i \leq i$ .

For example  $(2017)_{10} = (2, 4, 4, 0, 0, 1)_! = 2 \cdot 720 + 4 \cdot 120 + 4 \cdot 24 + 0 \cdot 6 + 0 \cdot 2 + 1 \cdot 1$ .

- i. Suggest an algorithm to convert from base-10 to the factoradic representation. What is the factoradic representation of  $(10000)_{10}$ ? What is the factoradic representation of  $(94356)_{10}$ ?
- ii. Here we explore addition in the factoradic system. Add the two numbers  $(4, 2, 2, 1)_!$  and  $(1, 3, 1, 1)_!$ , express the result in the factoradic representation. Do we need to perform the addition by first transforming the addends to the base-10 representation? Describe a general procedure for adding two numbers in the factoradic system.