Homework # 2

Due Tuesday, May 1, 2018, at 2:30 PM
Collaboration and discussions are not allowed on Problem 1
and are allowed and encouraged on Problem 2

1. **Positional Number Systems**(*Collaboration is not allowed on this Problem*)
Recall that an integer N represented in base-b is denoted by $N = (d_m, d_{m-1}, \ldots, d_1, d_0)_b$ with

$$N = \sum_{i=0}^{m} d_i \cdot b^i$$

and $0 \leq d_i \leq b - 1$.

(a) Given X and Y, two numbers with $m+1$ digits, represented in a base-b positional system. Namely, $X = (x_m, x_{m-1}, \ldots, x_1, x_0)_b$ and $Y = (y_m, y_{m-1}, \ldots, y_1, y_0)_b$. Prove the following statements for any base b.

i. $X = Y$ if and only if $x_i = y_i$ for all $0 \leq i \leq m$.

ii. Assume that $X \neq Y$. Suggest a simple algorithm to decide which number is bigger. *Hint:* Let j be the most significant position where X and Y differ. Prove that $X > Y$ if and only if $x_j > y_j$.

Reminder: Recall that an 'if and only if' (in short iff) proof requires proving the two directions of the claim; for instance, if we want to prove *(A iff B)*, then we must prove *(If A, then B)* and *(if B, then A)*.

(b) Prove the claims in (a) for X and Y two numbers with $m + 1$ digits, represented in a base-b no-0 positional system. *Before attempting this problem, it might be helpful to solve Problem 2.*

(c) Prove the claims in (a) for X and Y two numbers with m digits, represented in a factoradic number system. *Before attempting this problem, it might be helpful to solve Problem 2.*

(d) What are *all the possible* solutions for b? Show your work.

i. $(1234)_b + (5432)_b = (6666)_b$

ii. $(41)_b \div (3)_b = (13)_b$

iii. $\sqrt{(41)_b} = (5)_b$

(e) What are *all the possible* solutions for b and c? Show your work.

i. $(121)_b = (100)_c$

ii. $(50)_b = (43)_c$

2. **Strange and Beautiful Number Systems** (*Collaboration is allowed on this Problem*)

(a) **Base-b no-0 Positional Number System**

In class we studied the base-10 no-0 positional system where a positive integer N is represented as follows:

$$N = \sum_{i=0}^{m} d_i \cdot 10^i$$

with $d_i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, A = 10\}$ for all $0 \le i \le m$.

Examples, $N = (10)_{10} = (A)$, $N = (100)_{10} = (9A)$ and $N = 0 =$ (the blank string).

We expect you to solve the following problems with syntax-based procedures (as was demonstrated in class) and not by base-conversion procedures.

i. What is the base-10 no-0 representation of $(20006100101)_{10}$? Use the algorithm presented in class.

ii. Suggest an algorithm to convert from base-10 no-0 to base-10. Use your algorithm to find the base-10 representation of $(AA21A36A99A)$?

iii. Suggest an analogous base-2 no-0 representation.
Hint: the number $(4)_{10}$ is represented by $(100)_2$ in base-2 and by (12) in the base-2 no-0 representation.
Apply the analogous algorithms from parts (i) and (ii) to represent the base-2 number $(10110100100110110)_2$ in base-2 no-0 representation; and to represent the base-2 no-0 number (1122112121121112) in the base-2 representation.

(b) **Factoradic Number System**

A number N represented in the factoradic system is denoted by $N = (a_m, a_{m-1}, \ldots, a_2, a_1)_!$ with

$$N = \sum_{i=1}^{m} a_i \cdot i!$$

and $0 \le a_i \le i$.

For example $(2018)_{10} = (2, 4, 4, 0, 1, 0)_! = 2 \cdot 720 + 4 \cdot 120 + 4 \cdot 24 + 0 \cdot 6 + 1 \cdot 2 + 0 \cdot 1$.

i. Suggest an algorithm to convert from base-10 to the factoradic representation. What is the factoradic representation of $(10000)_{10}$? What is the factoradic representation of $(94356)_{10}$?

ii. Here we explore addition in the factoradic system. Add the two numbers $(4, 2, 2, 1)_!$ and $(1, 3, 1, 1)_!$, express the result in the factoradic representation. Do we need to perform the addition by first transforming the addends to the base-10 representation? Describe a general procedure for adding two numbers in the factoradic system.