at the centroid of three weights, f_a, f_m, and f_b, each suspended at the mid-point of its class. Finally, F_4 results from the assumption that the mode is equal to the abscissa of the maximum point on the parabola which passes through the three points, (x_a,f_a), $(x_m.f_m)$, and (x_b,f_b), where x_i denotes the mid-point of the particular class.

Graphical methods for obtaining F_4 from the histogram are well known. However, it does not seem to be as well known that the values of F_1, F_2, and F_3 may also be obtained by simple graphical methods. These methods are described below. Since the proofs require at most simple analytic geometry, they are omitted here.

The following figures are self-explanatory. In each case, only three rectangles of the histogram have been drawn. The construction for F_4 is included for completeness. It is of interest to note that the easiest geometric construction corresponds to the most complicated algebraic determination of the mode.

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A Number System Without a Zero-Symbol

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> "Conceived in all probability as a symbol for an empty column on a counting board, the Indian *sunya* was destined to become the turning point in a development without which the progress of modern science, industry, or commerce is inconceivable."—Tobias Dantzig in *Number, the Language of Science.*

Modern science, industry, and commerce have been possible because of an easily manipulated number system. This system is an historical consequence of the discovery of a zero-symbol. Does it follow, however, that an easily manipulated number system is impossible without the use of this symbol?

Consider a system (to be called the 0-less system) consisting of the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9, T in which the numerals 1 through 9 correspond to their counterparts in the decimal system and in which T has the same value as 10. In this system, the following sequence would exist:

$\cdots 9,\ T,\ 11 \cdots 19,\ 1T,\ 21 \cdots 99,\ 9T,\ T1,\ T2 \cdots T9,\ TT,\ 111 \cdots 999,\ 99T,\ 9T1,\ 9T2 \cdots 9T9,\ 9TT,\ T11 \cdots T99,\ T9T,\ TT1 \cdots TT9,\ TTT,\ 1{,}111 \cdots .$

Sequences in the 0-less system, it will be noted, follow the same general pattern as those in the conventional decimal system which do not include numbers having 0 as a component part. Sequences involving numbers including T are consequences of this pattern in the absence of a zero-symbol. Thus, the successor of T is 11, that of $9T$ is $T1$, that of TTT is 1,111. In general, in the successor number T is replaced by 1 and its predecessor numeral is increased in value by 1. The nth power of T consists of $(n-1)$ 9's followed by T. ($T^3=99T$; $T^4=999T$; $T^6=99999T$). The multiplication of any number by T^n (where n is a positive integer) is written by subtracting 1 from that number and following the difference with T^n expressed as a number. ($15\text{x}T=14T$; $15\text{ x }99T=1499T$; $TTT\text{ x }9T=TT99T$).

The foregoing characteristics of the 0-less system determine the operations involved in addition, subtraction, multiplication, and division. Comparative operations under the decimal and the 0-less system follow for illustrative purposes:

Addition:

Decimal	0-less
1,309	1,2T9
2,010	1,9TT
3,319	3,319

Subtraction:

Decimal	0-less
7,568	7,568
3,459	3,459
4,109	3,9T9

Multiplication:

Decimal	0-less
105	T5
246	246
630	62T
420	41T
210	1TT
25830	2582T

Division:

Decimal	0-less
246)25830(105	246)2582T(T5
246	245T
1230	122T
1230	122T

Since the multiplication of a number by T^n is indicated by subtracting 1 from the number and following the difference by T^n written as a number, the transformation of a fraction having T^n as a denominator to one in which the denominator is a greater power of T is a simple operation. $(17/9T = 16T/99T)$ As a consequence, the manipulations involving decimal notation are possible in the 0-less system. It is true that the absence of a zero-symbol precludes the use of the decimal point in writing fractions having values less than 0.1. Such fractions, however, can be written in the 0-less system in the form $m(T^{-n})$. Thus 0.015 in the conventional system is equivalent to $15(T^{-3})$ in the 0-less system. For convenience in adding decimal fractions in the 0-less system, the power of T of the largest valued fraction can be indicated, and the numerators of the others can be written as they would in the conventional system with the 0's following the decimal point eliminated. As an illustration, consider the addition of identical quantities in the conventional and the 0-less systems.

.719	$719(T^{-3})$
.0235	235
.604	$5T4$
.00173	173
1.34823	1.34823

It will be noted that it is not necessary in the 0-less system to transpose decimal fractions to the highest negative power of T as a preliminary step to addition.

The foregoing manipulations indicate that the 0-less system has substantially the elasticity of the conventional decimal system, and as a consequence challenges the assertion that modern science, inindustry, or commerce would be inconceivable without the zero-symbol, even though its discovery happened to be an historical condition to their development. While facility in ordinary arithmetic manipulations may not be dependent on the symbol, it must nevertheless be realized that the development of pure mathematics would have been retarded without it, since the study of classes necessitates the identification of the 0-class. This paper, therefore, is not to be interpreted as an argument that the values of the zero-symbol to mathematics are wholly accidental, but as a discussion of its alleged essential character in an easily manipulated system of numbers.