### IST 4: Planned Schedule - Spring 2015

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- **T** = today
- **x = hw#x out**
- **x = hw#x due**
- **oh = office hours**
- **Mx = MQx out**
- **Mx = MQx due**

**Midterms**

- Week 2: Midterms
Everything is 0-1 (Two Valued)

0-1 is an old idea, is a new idea, it is a good idea!

- A1. Identities:
  \[ a + 0 = a \quad \text{and} \quad a \cdot 1 = a \]

- A2. Complements:
  \[ a + \overline{a} = 1 \quad \text{and} \quad a \cdot \overline{a} = 0 \]

- A3. Commutativity:
  \[ a + b = b + a \quad \text{and} \quad a \cdot b = b \cdot a \]

- A4. Distributivity:
  \[ a + (b \cdot c) = (a + b) \cdot (a + c) \quad \text{and} \quad a \cdot (b + c) = (a \cdot b) + (a \cdot c) \]
Leibniz

Addition:

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When was the first binary adder built?

2 symbol adder

\[ \text{carry} = MAJ(d_1, d_2, c) \]

\[ \text{sum} = XOR(d_1, d_2, c) \]
The First Digital Adder
George Stibitz, 1904-1995

He worked at Bell Labs in New York

In the fall of 1937 Stibitz used surplus relays, tin can strips, flashlight bulbs, and other common items to construct his "Model K"

K stands for kitchen...
The First Digital Adder
George Stibitz, 1904-1995

• Bell Labs Model 1 Complex Calculator
• 450 relays
• Remote operation... Via telegraph
Logic to Physics

Claude Shannon
Claude Elwood Shannon was born in Petoskey, Michigan, on April 30, 1916. The first sixteen years of Shannon's life were spent in Gaylord, Michigan.
Shannon's Background

Claude Elwood Shannon was born in Petoskey, Michigan, on April 30, 1916.

In 1932 (16) he entered the University of Michigan, where he took a course that introduced him to the work of George Boole.

He graduated in 1936 (20) with two bachelor's degrees, one in electrical engineering and one in mathematics.

Joined MIT in 1936, received the masters in electrical engineering and a doctorate in Mathematics, at the 1940 (24) commencement.
Shannon’s Inspiration
Joined MIT in 1936

Vannevar Bush 1890 - 1974
Samuel Caldwell 1904-1960

The differential analyzer at MIT (1931) was the first general equation solver. It could handle sixth-order differential equations.
Connection Between
Boolean Calculus and Physical Circuits
Shannon 1938

Shannon
1916-2001

Hitchcock
1875-1957

A Symbolic Analysis of Relay and Switching Circuits*

Claude E. Shannon**

Shannon's advisor both MSc and PhD - a mathematician


** Claude E. Shannon is a research assistant in the department of electrical engineering at Massachusetts Institute of Technology, Cambridge. This paper is an abstract of a thesis presented at MIT for the degree of master of science. The author is indebted to Doctor F. L. Hitchcock, Doctor Vannevar Bush, and Doctor S. H. Caldwell, all of MIT, for helpful encouragement and criticism.

77 years ago
Hitchcock 1875-1957

Bush 1890-1974

Sutherland was faculty at Caltech from 1974 to 1978 (also MS degree)

Served as the founding chair of the CS Department at Caltech

Shannon 1916-2001

Ivan Sutherland 1938-
Connection Between
Boolean Calculus and Physical Circuits
Shannon 1938

A Symbolic Analysis of Relay and Switching Circuits*

Claude E. Shannon**

In the control and protective circuits of complex electrical systems it is frequently necessary to make intricate interconnections of relay contacts and switches. Examples of these circuits occur in automatic telephone exchanges, industrial motor-control equipment, and in almost any circuits designed to perform complex operations automatically. In this paper a mathematical analysis of certain of the properties of such networks will be made. Particular attention will be given to the problem of network synthesis.

No mention of computers.... they did not exist

A concept that is missing in the text?
Logic to Physics

The language of lines
Boolean Calculus and Physical Circuits
Single Lines and Composition

The language of lines:
A line can have only two possible colors: blue or red

Two lines can be composed in two possible ways

In parallel:

In series:
Boolean Calculus and Physical Circuits
Endpoints

Lines have endpoints
Compositions have endpoints

Two lines can be composed in two possible ways

In parallel:
Lines have endpoints

Compositions have endpoints

Two lines can be composed in two possible ways

In series:
Compositions of lines can be composed in two different ways, using their endpoints.

In parallel:
Compositions of lines can be composed in two different ways, using their endpoints.

In parallel:
Compositions of lines can be composed in two different ways, using their endpoints.

In series:

- a → b

- a → b
Compositions of lines can be composed in two different ways, using their endpoints.

In series:
What is the color of a composition?

- color = blue
- color = red

???
What is the color of a composition?

The color of a composition is red if there is a red path between the endpoints. Otherwise, the color is blue.
The **two-color** line composition is a $0-1$ Boolean algebra!

How can we prove it?
Boolean Algebra

An algebraic system \( B \), set of elements \( B \),
two binary operations \( + \) and \( \cdot \).
\( B \) has at least two elements (0 and 1).

If the following axioms are true
then it is a Boolean Algebra:

A1. identity
\[ a + 0 = a \quad \text{and} \quad a \cdot 1 = a \]

A2. complement
\[ a + \bar{a} = 1 \quad \text{and} \quad a \cdot \bar{a} = 0 \]

A3. commutative
\[ a + b = b + a \quad \text{and} \quad a \cdot b = b \cdot a \]

A4. distributive
\[ a + b \cdot c = (a + b) \cdot (a + c) \]
\[ a \cdot (b + c) = a \cdot b + a \cdot c \]
Two-Colored Line Composition and 0-1 Boolean Algebra

Algebraic system: set of elements $B$, two binary operations $+$ and $\cdot$. $B$ has at least two elements (0 and 1)

Elements:

0

1
Two-Colored Line Composition and 0-1 Boolean Algebra

**Algebraic system:** set of elements $B$, two binary operations $+$ and $\cdot$. $B$ has at least two elements (0 and 1)

**Elements:**

The color of a composition is red if there is a red path between the endpoints. Otherwise, the color is blue.

**Operations:**

**Compose in parallel:** $+$

**Compose in series:** $\cdot$
Two-Colored Line Composition and 0-1 Boolean Algebra

The color of the composition equals the color of \( a \)

The color of the composition equals the color of \( a \)

Compose in parallel

Compose in series

The color of a composition is red if there is a red path between the endpoints. Otherwise, the color is blue.
Two-Colored Line Composition and 0-1 Boolean Algebra

- Compose in series
  - The color of the composition is **red** if there is a red path between the endpoints. Otherwise, the color is **blue**

- Compose in parallel

- A1. Identities:
  \[ a + 0 = a \text{ and } a \cdot 1 = a \]

- A2. Complements:
  \[ a + a = 1 \text{ and } a \cdot a = 0 \]

- A3. Commutativity:
  \[ a + b = b + a \text{ and } a \cdot b = b \cdot a \]

- A4. Distributivity:
  \[ a \cdot (b \cdot c) = (a + b) \cdot (a + c) \text{ and } a \cdot (b + c) = (a \cdot b) + (a \cdot c) \]

The color of the composition is **red** = 1

The color of the composition is **blue** = 0
Two-Colored Line Composition and 0-1 Boolean Algebra

- **A1. Identities:**
  \[ a + 0 = a \quad \text{and} \quad a \cdot 1 = a \]

- **A2. Complements:**
  \[ a + \bar{a} = 1 \quad \text{and} \quad a \cdot \bar{a} = 0 \]

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- **A4. Distributivity:**
  \[ a + (b \cdot c) = (a + b) \cdot (a + c) \quad \text{and} \quad a \cdot (b + c) = (a \cdot b) + (a \cdot c) \]

By the definition of the color of a composition:

- **Compose in parallel**
- **Compose in series**

The **color** of a composition is **red** if there is a red path between the endpoints. Otherwise, the **color is blue**.
Two-Colored Line Composition and 0-1 Boolean Algebra

Two paths: ab and ac

- A1. Identities:
  \[ a + 0 = a \] and \[ a \cdot 1 = a \]

- A2. Complements:
  \[ a + \bar{a} = 1 \] and \[ a \cdot \bar{a} = 0 \]

- A3. Commutativity:
  \[ a + b = b + a \] and \[ a \cdot b = b \cdot a \]

- A4. Distributivity:
  \[ a + (b \cdot c) = (a + b) \cdot (a + c) \] and \[ a \cdot (b + c) = (a \cdot b) + (a \cdot c) \]
Is the **two-color** line composition a **0-1** Boolean algebra?

**YES**
“We are now in a position to demonstrate the equivalence of this calculus with certain elementary parts of the calculus of propositions.”

“The algebra of logic originated by George Boole, is a symbolic method of investigating logical relationships.”

“The symbols of Boolean algebra admit of two logical interpretations. If interpreted in terms of classes, the variables are not limited to the two possible values 0 and 1.”

“E. V. Huntington' gives the following set of postulates for symbolic logic:”
Shannon used **relays** and connected them in **series-parallel circuits**

A Symbolic Analysis of Relay and Switching Circuits*

*Claude E. Shannon**

**Relay on the edge controlled by a 0-1 variable**
Connection Between Boolean Calculus and Physical Circuits
Shannon 1938

A Symbolic Analysis of Relay and Switching Circuits

Claude E. Shannon

Shannon 1916-2001

The value of a circuit is 1 if there is a connected path between the endpoints. Otherwise, it is 0.

The color of a composition is red if there is a red path between the endpoints. Otherwise, the color is blue.
Relay Circuits
analysis and synthesis
In Shannon’s words:

“..any circuit is represented by a set of equations, The terms of the equations corresponding to the various relays and switches in the circuit.”

“A calculus is developed for manipulating these equations by simple mathematical processes most of which are similar to ordinary algebraic algorisms.”

“This calculus is shown to be exactly analogous to the calculus of propositions used in the symbolic study of logic.”
In Shannon’s words:

“For the synthesis problem the desired characteristics are first written as a system of equations, and the equations are then manipulated into the form representing the simplest circuit. The circuit may then be immediately drawn from the equations.”
For the synthesis problem the desired characteristics are first written as a system of equations, and the equations are then manipulated into the form representing the simplest circuit. The circuit may then be immediately drawn from the equations.

The language of Logic Design is born!
Relay Circuits analysis
A relay circuit corresponds to a formula

relay circuits  →  Boolean functions

Boolean sum of all the paths between endpoints
Analysis of Relay Circuits

Example 1:
- series-parallel
- independent paths between endpoints
Example 1:
- series-parallel
- independent paths between endpoints

\[ f(a, b, c, d, e) = a \cdot b + d \cdot e + a \cdot c \cdot e + b \cdot c \cdot d \]
Example 1:
- series-parallel
- independent paths between endpoints

\[ f(a, b, c, d, e) = a \cdot b + d \cdot e + a \cdot c \cdot e + b \cdot c \cdot d \]
Example 1:
- series-parallel
- independent paths between endpoints

\[ f(a, b, c, d, e) = a \cdot b + d \cdot e + a \cdot c \cdot e + b \cdot c \cdot d \]
Example 1:
- series-parallel
- independent paths between endpoints

\[ f(a, b, c, d, e) = a \cdot b + d \cdot e + a \cdot c \cdot e + b \cdot c \cdot d \]
Example 2:
- non series-parallel
- dependent paths between endpoints
Analysis of Relay Circuits

\[ f(a, b, c, d, e) = a \cdot b + d \cdot e + a \cdot c \cdot e + b \cdot c \cdot d \]
Analysis of Relay Circuits

\[ f(a, b, c, d, e) = a \cdot b + d \cdot e + a \cdot c \cdot e + b \cdot c \cdot d \]
Analysis of Relay Circuits

\[ f(a, b, c, d, e) = a \cdot b + d \cdot e + a \cdot c \cdot e + b \cdot c \cdot d \]
Analysis of Relay Circuits

\[ f(a, b, c, d, e) = a \cdot b + d \cdot e + a \cdot c \cdot e + b \cdot c \cdot d \]
Analysis of Relay Circuits

Example 3: multiple terminals (also in HW#4)

How many functions?
Analysis of Relay Circuits

Example 3:
multiple terminals (also in HW#4)

\[ f(a, b, c, d, e) = a \cdot b + d \cdot e + a \cdot c \cdot e + b \cdot c \cdot d \]
Analysis of Relay Circuits

Example 3: multiple terminals (also in HW#4)

\[ a \cdot d + c + b \cdot e \]
Analysis of Relay Circuits

Example 3:
multiple terminals (also in HW#4)

\[ a \cdot d + c + b \cdot e \]
Analysis of Relay Circuits

Example 3: multiple terminals (also in HW#4)

\[ a \cdot d + c + b \cdot e \]
Analysis of Relay Circuits

Example 3: multiple terminals (also in HW#4)

\[ b + c \cdot e + a \cdot d \cdot e \]
Analysis of Relay Circuits

Example 3: multiple terminals (also in HW#4)

\[ b + c \cdot e + a \cdot d \cdot e \]
Analysis of Relay Circuits

Example 3: multiple terminals (also in HW#4)

\[ b + c \cdot e + a \cdot d \cdot e \]
Analysis of Relay Circuits

Example 4:
- series-parallel
- MANY dependent paths between endpoints

Q: how many FORWARD paths?
Analysis of Relay Circuits

Example 4:
- series-parallel
- MANY dependent paths between endpoints

Q: how many FORWARD paths? $3 \times 3 \times 3 \times 3 = 81$
Analysis of Relay Circuits

Example 4:
- series-parallel
- MANY dependent paths between endpoints

Red $a =$ variable $a$

Blue $a =$ complement of $a$

Diagram of a relay circuit showing multiple paths and nodes labeled with $a$, $b$, $c$, $d$, and $e$. The circuit is depicted with red and blue lines, indicating variable $a$ and its complement, respectively.
Example 4:
- series-parallel
- MANY dependent paths between endpoints

Red $a =$ variable $a$

Blue $a =$ complement of $a$

Q: Is $(a=0, b=1, c=1, d=1, e=1)$ a satisfying assignment?

NO
Analysis of Relay Circuits

Example 4:
- series-parallel
- MANY dependent paths between endpoints

Red $a = \text{variable } a$

Blue $a = \text{complement of } a$

Q: Is $(a=1, b=1, c=1, d=1, e=1)$ a satisfying assignment? **YES**
Analysis of Relay Circuits

Is there a satisfying assignment?
Analysis of Relay Circuits

Example 5:
- series-parallel
- MANY dependent paths between endpoints

Red $a =$ variable $a$

Blue $a =$ complement of $a$

Q: Is there a satisfying assignment?

? NO
Analysis of Relay Circuits

Example 5:
- series-parallel
- MANY dependent paths between endpoints

Red \( a = \) variable \( a \)

Blue \( a = \) complement of \( a \)

\[ a \]
\[ b \]
\[ c \]
\[ a \]
\[ b \]
\[ c \]

Q: Is there a satisfying assignment? 

\( b \) must be 1
Analysis of Relay Circuits

Example 5:
- series-parallel
- MANY dependent paths between endpoints

Red $a = \text{variable } a$

Blue $a = \text{complement of } a$

Q: Is there a satisfying assignment?

$\textbf{Contradiction!}$

$b$ must be 0
Efficient algorithms?

Questions on satisfying (SAT) assignments?

Is a given assignment satisfying?

Is there a satisfying assignment?
There is an efficient algorithm for verifying a given SAT solution for any structure!

It is related to algorithms for solving connectivity problems in graphs.
Algoritms for **finding** a SAT assignment?

If the circuit has width **2**: There is an efficient algorizm for finding a satisfying assignment...
However, if the circuit has **width 3**: 
• No efficient algorizm is known! 
• Likely, an efficient algorizm does not exist
Efficient algorithms?

Questions on satisfying (SAT) assignments?
- Is a given assignment satisfying?
- Is there a satisfying assignment?

Shannon’s connection between computation and Boolean algebra is at the core of Algorithms and Complexity! “P vs NP question”
What is the function?
What is the function?

Red $a = \text{variable } a$

Blue $a = \text{complement of } a$
What is the function?

Red $a = \text{variable } a$

Blue $a = \text{complement of } a$

odd parity

even parity

The key: 1 causes a switch in parity
0 keeps the parity the same
What is the function?

**odd parity**

- $a=1$
- $b=1$
- $c=0$
- $d=1$

**even parity**

The key: 1 causes a switch in parity 0 keeps the parity the same
What is the function?

Red $a =$ variable $a$

Blue $a =$ complement of $a$

odd parity

even parity

The key: 1 causes a switch in parity
0 keeps the parity the same

$XOR(a, b, c, d) = a \oplus b \oplus c \oplus d$
THE DESIGN OF SWITCHING CIRCUITS

BY

WILLIAM KEISTER

ALISTAIR E. RITCHIE

SETH H. WASHBURN

MEMBERS OF THE TECHNICAL STAFF
BELL TELEPHONE LABORATORIES

Third Printing
William Keister was a pioneer in switching theory and design at Bell Labs when he retired in 1972, he was director of Bell Labs' Computing Technology Center at Holmdel, New Jersey.

Keister began working in his spare time to prove that puzzles could be solved using Boolean algebra.

U.S. Patent 3637216 (1972): The Hexadecimal Puzzle
The First Book on Switching Circuits
Keister, Ritchie and Washburn, 1951

THE DESIGN OF SWITCHING CIRCUITS

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BELL TELEPHONE LABORATORIES

Third Printing

C and Unix
Dennis Ritchie
1941-2011
Son of
The First Book on Switching Circuits
Keister, Ritchie and Washburn, 1951

Being recognized by the president
with co-inventor Ken Thompson

C and Unix
Dennis Ritchie
1941-2011

Son of
This book is not a text on telephone systems. It is concerned, rather, with the basic techniques of switching circuit design: techniques which are applicable to digital computers and other complex control systems as well as telephone switching systems. The writing of this text was started soon after the end of World War II as a series of lecture notes for use in training new engineers in the Switching Systems Development Department of the Bell Telephone Laboratories. During the succeeding years the notes were revised and the methods of instruction were improved. In 1950, with general interest in the subject of switching increasing, arrangements were made with the Massachusetts Institute of Technology to give to graduate students a one-semester course in switching circuit design. The text for this course was based on the material used for training within the Laboratories. The present volume is a final edition of the text used in the M. I. T. course, revised to take advantage of the academic experience gained there.
Homework #4

Due Thursday, May 21, 2015, at 2:30 PM

Collaboration and discussions are not allowed on Problem 1
and are allowed and encouraged on Problem 2

**Important note**

**Shannon Used the Dual Notation**

In this HW set use the notation from class!

1 = closed relay/circuit
0 = open relay/circuit
1. **A Generalization to Multiple Terminals** (*Collaboration is not allowed on this Problem*)

A generalization of the switching (relay) circuit model is a circuit with multiple terminals. The Boolean function $X_{ab}$ is 1 if there is a closed path between terminals $a$ and $b$, and 0 otherwise. With multiple terminals, $a, b, c, d, \ldots$, Boolean functions exist between any pair of terminals. For example, for the circuit

![Circuit Diagram]

we have

\[
X_{af} = x \cdot y \cdot z \\
X_{bd} = \bar{x} \cdot z \\
X_{cf} = 0
\]

and so on.
1. **A Generalization to Multiple Terminals** *(Collaboration is not allowed on this Problem)*

A generalization of the switching (relay) circuit model is a circuit with multiple terminals. The Boolean function $X_{ab}$ is 1 if there is a closed path between terminals $a$ and $b$, and 0 otherwise. With multiple terminals, $a, b, c, d, \ldots$, Boolean functions exist between any pair of terminals. For example, for the circuit

![Diagram of a circuit with multiple terminals](image)

we have

\[
X_{af} = x \cdot y \cdot z
\]
\[
X_{bd} = \bar{x} \cdot z
\]
\[
X_{cf} = 0
\]

and so on.
One circuit with multiple terminals for many functions

(a) Construct a circuit with 3 relays that implements the functions

\[ f_1 = x \cdot y \]
\[ f_2 = \bar{x} \cdot y \]

(b) Construct a circuit with 4 relays that implements the functions

\[ f_1 = x \cdot y + \bar{x} \cdot \bar{y} \]
\[ f_2 = \bar{x} \cdot y + x \cdot \bar{y} \]
One circuit with multiple terminals for many functions

(c) Construct a circuit with 6 relays that implements the functions:

\[ f_1 = x \cdot (y + z) \]
\[ f_2 = y \cdot (x + z) \]
\[ f_3 = z \cdot (x + y) \]
\[ f_4 = x + y \cdot z \]
\[ f_5 = y + x \cdot z \]
\[ f_6 = z + x \cdot y \]

(d) (Extra Credit: 10% of the total homework grade). Construct a circuit with as few relays as possible that implements all 16 functions of two variables. A solution with 8 relays gets full credit.
2. The Complete Quadratic Function \( \text{(Collaboration is allowed on this Problem)} \)

The Complete Quadratic (CQ) function is the Boolean function that consists of the XOR of all the \( \binom{n}{2} \) possible AND’s between pairs of inputs. Namely,

\[
CQ(X) = (x_1 \cdot x_2) \oplus (x_1 \cdot x_3) \oplus \cdots \oplus (x_{n-1} \cdot x_n).
\]

For example,

\[
CQ(x_1, x_2, x_3) = (x_1 \cdot x_2) \oplus (x_1 \cdot x_3) \oplus (x_2 \cdot x_3).
\]

Wait until after the class on Thursday....

(a) In class we proved that a Boolean function \( f(X) \) is symmetric iff it is a function of \( |X| \) (the number of 1s in \( X \)). Write \( CQ(X) \) with 6 inputs as a function of \( |X| \). Note that there are 7 entries in the table.

(b) Generalization: For an arbitrary \( n \), express \( CQ(X) \) as a function of \( |X| \). Namely, you need to specify, as a mathematical expression, the values of \( |X| \) for which \( CQ(X) = 1 \). Justify your solution.
2. **The Complete Quadratic Function** (*Collaboration is allowed on this Problem*)

The Complete Quadratic (CQ) function is the Boolean function that consists of the XOR of all the \( \binom{n}{2} \) possible AND’s between pairs of inputs. Namely,

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CQ(X) = (x_1 \cdot x_2) \oplus (x_1 \cdot x_3) \oplus \cdots \oplus (x_{n-1} \cdot x_n).
\]

For example,

\[
CQ(x_1, x_2, x_3) = (x_1 \cdot x_2) \oplus (x_1 \cdot x_3) \oplus (x_2 \cdot x_3).
\]

*Wait until after the class on Thursday....*

(c) Design and draw a switching circuit for \( CQ(X) \) with five inputs, namely, \( X \in \{0, 1\}^5 \). Please use as few relays as possible. A solution with 20 relays (or less) gets full credit. Justify your solution.

*Hint:* Use the construction of XOR from class as an inspiration.